IX-7. A COUPLED-MODE DESCRIPTION OF THE REGGIA-SPENCER PHASE SHIFTER

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Introduction - Although Reggia-Spencer phase shifters have been built for many years, no completely satisfactory description of the principle of operation of this type of device has appeared in the literature. Of the many theories advanced to explain the phase shifter operation, two have retained some measure of vogue. Button and Lax[1] have postulated that the basic mechanism is the change in effective permeability with applied field, enhanced by the dielectric loading effects of the ferrite. Their view is supported by J. A. Weiss[2], who identified frequency-periodic resonances as the onset of Faraday rotation in a rectangular guide structure. Rizzi and Gatlin[3] suggested that the ferrite rod acts as a "dielectric waveguide" and that the phase shift results from Faraday rotation of the r-f fields with applied field. Subsequent computations by Tompkins, Reggia, and Joseph[4] have tended to verify this point of view.

These two schools of thought are not only contradictory, but suffer from a failure to account for experimentally observed behavior of practical devices. The "effective permeability" argument begins by assuming that the permeability of an unmagnetized ferrite is unity, an assumption that needs proving in the region just above $\omega = \omega_m$ commonly used in Reggia-Spencer devices. Worse yet, this theory predicts a decrease of β with applied field, which is exactly opposite to measured data on practical phase shifters of the Reggia-Spencer type. Other experimental anomalies exist, including the failure to account for the threshold condition for large increase of phase shift with rod diameter, so that Lax and Button[5] have concluded that coupling between vertically and horizontally polarized TE modes is an important factor for rods of large diameter. On the other hand, candid acceptance of the Faraday rotation viewpoint runs aground immediately; there simply does not seem to be any rational explanation for the way the structure maintains proper polarization at its output and provides reciprocal insertion phase at arbitrary levels of applied field between zero and saturation. The frequency-periodic resonance behavior noted by Weiss would seem to be more nearly characteristic for a structure permitting Faraday rotation.

The objective of this paper is to present an alternate point of view for explaining the operation of the Reggia-Spencer phase shifter. The fundamental postulate is that the basic mechanism of this reciprocal phase shifter is a nonreciprocal Faraday-effect coupling of the dominant vertically polarized TE mode to the cross-polarized TE mode in rectangular waveguide. In the frequency range where large phase shift is obtained, Faraday rotation occurs, even in the presence of an applied longitudinal magnetic field. However, a substantial reciprocal increase in the phase factor of the perturbed dominant mode takes place because of the mode coupling through the ferrite by an applied field. The frequency-periodic resonances identified by Weiss as being caused by Faraday rotation are thus seen to be the logical consequence when the frequency is raised above the cutoff point of the horizontally polarized mode. Analysis of this coupled-mode system can be made on the basis of general transmission line theory, taking into account the possibility of gyromagnetic coupling. The propagation factor for determining the insertion phase of the device then appears as one of the normal-mode propagation factors of the coupled-line structure.

Normal-mode Analysis - Consider two generalized uniform transmission lines, e.g. coaxial lines, waveguides, or, in particular, two nominally orthogonal modes within the same structure, and designate the "voltages" and "currents" of these lines by V_1, V_2 and I_1, I_2 respectively. Note that in the case of waveguide modes, these quantities need only be the longitudinally dependent complex amplitudes of suitably defined normalized mode functions for the transverse electric and magnetic field intensities. Associated with these lines will be the propagation factors γ_{01}, γ_{02} as well as the characteristic impedances Z_{01}, Z_{02}. The longitudinal variation of the field amplitude quantities is given by the familiar telegrapher's equations,

$$\frac{dV_1}{dz} = -\gamma_{01}Z_{01}I_1 \qquad \frac{dV_2}{dz} = -\gamma_{02}Z_{02}I_2 \tag{1 \& 2}$$

$$\frac{dI_1}{dz} = -\frac{\gamma_{01}}{Z_{01}}V_1 \qquad \frac{dI_2}{dz} = -\frac{\gamma_{02}}{Z_{02}}V_2 \tag{3 \& 4}$$

Now consider the effects of a nonreciprocal coupling ζ between the lines, in particular gyromagnetic coupling. It is appropriate to describe[6] such a coupling by means of the matrix differential equation,

$$\begin{bmatrix} \frac{dV_1}{dz} \\ \frac{dV_2}{dz} \end{bmatrix} = \frac{1}{1-\zeta^2}\begin{bmatrix} \gamma_{01}Z_{01} & j\zeta\sqrt{\gamma_{01}Z_{01}\gamma_{02}Z_{02}} \\ -j\zeta\sqrt{\gamma_{01}Z_{01}\gamma_{02}Z_{02}} & \gamma_{02}Z_{02} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{5}$$

Equations (3) and (4) may also be combined formally to give the matrix relation,

$$\begin{bmatrix} \frac{dI_1}{dz} \\ \frac{dI_2}{dz} \end{bmatrix} = \begin{bmatrix} \frac{\gamma_{01}}{Z_{01}} & 0 \\ 0 & \frac{\gamma_{02}}{Z_{02}} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{6}$$

By analogy with the usual method of solution for the telegrapher's equations, equation (5) is differentiated with respect to the longitudinal coordinate z, and the value of the column vector for the current derivatives from equation (6) is substituted into the right-hand side.

Solutions are then assumed that vary as $e^{-\gamma z}$ for V_1 and V_2, and substituted accordingly for the second derivatives of voltage with respect to z. A matrix eigenvalue results, with the secular equation,

$$\gamma^4 - \frac{\left(\gamma_{01}^{\ 2} + \gamma_{02}^{\ 2}\right)}{1 - \zeta^2}\ \gamma^2 + \gamma_{01}^{\ 2}\gamma_{02}^{\ 2}\ \frac{\left(1 - \zeta^2\right)}{\left(1 - \zeta^2\right)^2} = 0 \qquad (7)$$

with the roots for γ having positive z-progression being

$$\gamma = \frac{1}{\sqrt{2\ (1 - \zeta^2)}}\left[\left(\gamma_{01}^{\ 2} + \gamma_{02}^{\ 2}\right) \pm \left(\gamma_{01}^{\ 2} - \gamma_{02}^{\ 2}\right)\sqrt{1 + \frac{4\gamma_{01}^{\ 2}\gamma_{02}^{\ 2}\zeta^2}{\left(\gamma_{01}^{\ 2} - \gamma_{02}^{\ 2}\right)^2}}\ \right]^{1/2} \qquad (8)$$

This relationship indicates two values for the normal-mode propagation factors of the coupled-line system. The root using the positive sign is the perturbed value of γ_{01}, while choosing the negative sign in equation (8) gives the root which represents the perturbation of γ_{02} by the coupling. In the absence of coupling ($\zeta = 0$) it is clear that the roots of equation (8) are γ_{01} and γ_{02}. Finally, it is possible to substitute the roots γ_1, γ_2 of (8) back into the characteristic equation and construct the eigenvectors appropriate to the normal-mode propagation factors. For γ_{01} and γ_{02} relatively dissimilar and for modest amounts of coupling, the eigenvalues associated with γ_1 and γ_2 will show most of the field energy in lines 1 and 2, respectively.

These results have an important bearing on the Reggia-Spencer phase shifter. The large amount of phase shift, slope of the phase shift vs. frequency, and dependence of phase shift on bias field and cross-sectional geometry for this device give strong indication that the primary mechanism producing phase shift is a magnetic-field controlled coupling between the propagating dominant TE_{10}-type mode and an evanescent TE_{01}-type mode. Designating the propagating mode as line 1 and the cutoff mode as line 2, the conditions on γ_{01} and γ_{02} are:

$$\begin{aligned} \gamma_{01} &= j\beta_{01} \\ \gamma_{02} &= \alpha_{02} \end{aligned} \qquad (9)$$

with Z_{01} real and Z_{02} imaginary. Substituting into equation (8) and selecting the positive root, the perturbed value of γ_{01} will be:

$$\gamma_1 = \frac{j\beta_{01}}{\sqrt{2(1 - \zeta^2)}}\left[\left(1 - r^2\right) + \sqrt{\left(1 + r^2\right)^2 - 4r^2\zeta^2}\ \right]^{1/2} \qquad (10)$$

where $r \equiv \frac{\alpha_{02}}{\beta_{01}}$. This relationship expresses the dependence of the sought-after change in dominant-mode propagation factor as a function of the propagation constants of the unperturbed TE_{10} and TE_{01}-type modes and of the field-variable gyromagnetic coupling through the ferrite. It is fundamental in the sense that no particular geometry has been assumed for the guide cross-section, and no restrictions have been placed on the permissible values of r and ζ. Hence it is meaningful to explore the implications of this result with respect to characteristic behavior of Reggia-Spencer phase shifters of arbitrary design. To put equation (10) into more convenient form, define

$$\frac{\Delta\phi}{\phi_o} = \frac{\Delta\beta}{\beta_{01}} = \frac{\beta_1 - \beta_{01}}{\beta_{01}} = \frac{\beta_1}{\beta_{01}} - 1 = \left[\frac{\left(1 - r^2\right) + \sqrt{\left(1 + r^2\right)^2 - 4r^2\zeta^2}}{2\left(1 - \zeta^2\right)}\right]^{1/2} - 1 \qquad (11)$$

where $\Delta\phi$ is the phase shift available from a given coupled length, and ϕ_o is the insertion phase of this same section with no applied field.

A family of curves of $\Delta\phi/\phi_o$ has been plotted in Figure 1. These curves suggest the sort of variation of phase shift vs. frequency characteristics observed experimentally for this general type of coupling interaction. For a quantitative comparison with a practical phase shifter, it is necessary to specify the manner in which r varies with frequency for the device in question. This is an involved problem for a composite structure such as the Reggia-Spencer device. For the sake of simplicity, the linear assumption $r = A\ (f_{c2} - f)$, was made, and the scale factor A was chosen to fit experimental data at 8.5 Gc on a phase shifter whose f_{c2} was approximately 10 Gc. The theoretical curve for phase shift vs. frequency, adjusted to agree with measured data at 10.0 Gc, is plotted along with experimental data points in Figure 2.

Conclusions - Despite the crude assumption about the variation of the parameter r with frequency, a fairly good agreement appears to exist between the shapes of the theoretical and experimental curves. Therefore it can be concluded that the coupled-mode model is potentially capable of explaining the phase shift vs. frequency characteristics of the Reggia-Spencer phase shifter. Furthermore, this model avoids the difficulties of previous explanations with regard to direction of phase shift (β increases with ζ) and lack of Faraday rotation in the resonance-free region. From these and other observed characteristics, it appears that a very strong case can be made for mode coupling as the dominant mechanism producing phase shift in the Reggia-Spencer phase shifter.

References

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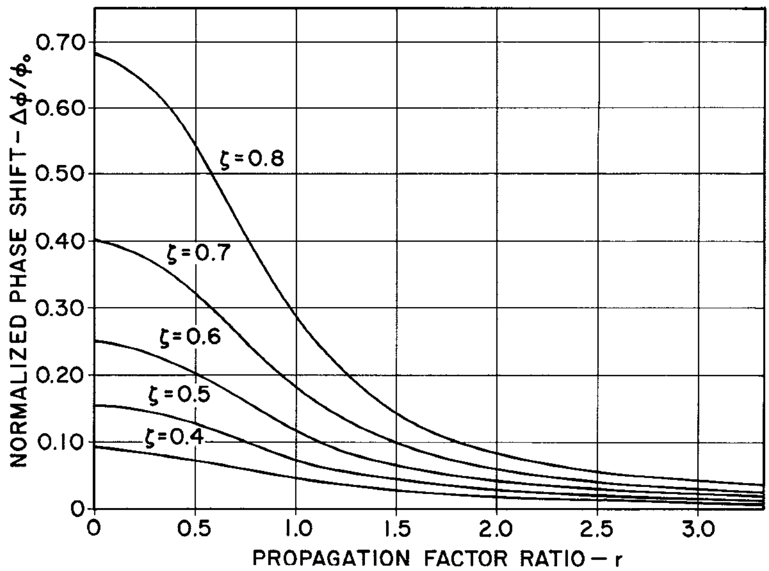

Figure 1. Relative Phase Shift as a Function of TE_{01}-Mode Proximity to Cutoff, for Selected Faraday-Effect Coupling Values

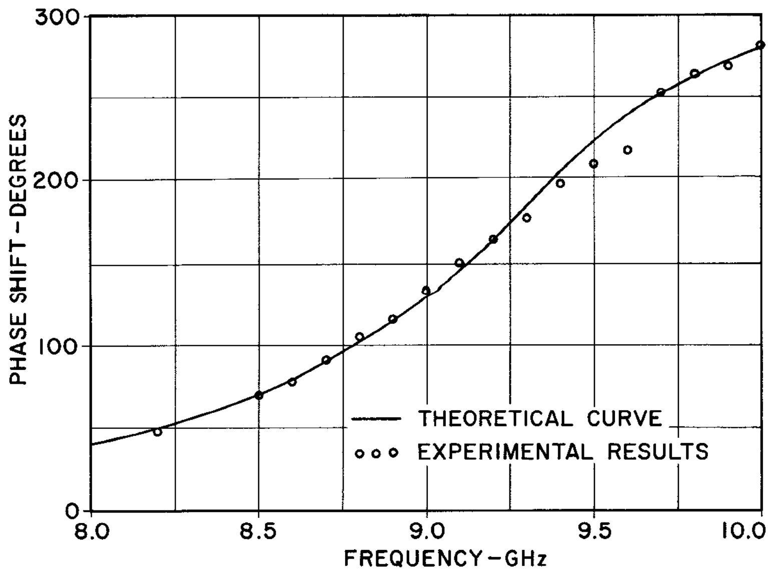

Figure 2. Comparison of Theoretical and Experimental Results on a Rantec Model PX-212 Reggia-Spencer Phase Shifter